JHEP

Published by Institute of Physics Publishing for SISSA

Received: *November 1, 2005*
Accepted: *December 14, 2005*
Published: *January 4, 2006*

Analytical tachyonic lump solutions in open superstring field theory

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Abstract: We construct a classical solution in the GSO(−) sector in the framework of a Wess-Zumino-Witten-like open superstring field theory on a non-BPS D-brane. We use an $su(2)$ supercurrent, which is obtained by compactifying a direction to a circle with the critical radius, in order to get analytical tachyonic lump solutions to the equation of motion. By investigating the action expanded around a solution we find that it represents a deformation from a non-BPS D-brane to a D-brane-anti-D-brane system at the critical value of a parameter which is contained in classical solutions. Although such a process was discussed in terms of boundary conformal field theory before, our study is based on open superstring field theory including interaction terms.

Keywords: String Field Theory, Tachyon Condensation, D-branes.

JHEP01(2006)013

Contents

1. Introduction

Analytic classical solutions have been found in open superstring field theory [1 – 3] on BPS D-branes formulated in terms of the Wess-Zemuno-Witten (WZW) like action [4, 5].[1] The solutions in ref. [1] are constructed from supercurrents, ghost fields and the identity string field. The characteristic features of the solution are its correspondence to the marginal deformation generated by the supercurrent and a well-defined Fock space expression of the solution.

In open bosonic string field theory, similar classical solutions have been constructed by using currents [1, 8 – 12]. They also correspond to marginal deformations and have a well-defined Fock space expression. Unfortunately, the vacuum energy of the bosonic solution is provided as a kind of indefinite quantities. In the absence of appropriate regularization, we have nothing else to do but evaluate it by indirect calculation. However, the remarkable feature of the supersymmetric solutions is that their vacuum energy vanishes exactly by a direct calculation [1] as expected from their correspondence to marginal deformations [13]. The supersymmetric case may provide a clue for solving the vacuum energy problem in the bosonic case.

Even on non-BPS D-branes, we can formulate string field theory in terms of the WZW like action [5, 14]. Since non-BPS D-branes have tachyonic modes in the GSO(−) sector, the theory enables us to investigate D-brane decay processes. A tachyonic lump solution, for instance, describes a deformation from a non-BPS D-brane to a D-brane-anti-D-brane pair. Actually, several analyses were performed by using the level truncation scheme [14 – 17]. If one of the directions is compactified on the circle with the critical radius, the above

[1] A string field theory around the solutions in ref. [2] was analyzed in refs. [6, 7]

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process is realized by a marginal deformation [18]. Accordingly, we have only to extend the solution on the BPS D-brane to the non-BPS case in order to construct the solution corresponding to the tachyonic lump. In this paper, we will construct an analytical solution of this tachyonic lump.

In open superstring field theory on a single non-BPS D-brane, the action of the NS sector string field is given by [5, 14]

$$S[\hat{\Phi};\hat{Q}_{\rm B}] = \frac{1}{4g^2}\Big\langle\Big\langle (e^{-\hat{\Phi}}\hat{Q}_{\rm B}e^{\hat{\Phi}})(e^{-\hat{\Phi}}\hat{\eta}_0 e^{\hat{\Phi}}) - \int_0^1 dt\, (e^{-t\hat{\Phi}}\partial_t e^{t\hat{\Phi}})\left\{(e^{-t\hat{\Phi}}\hat{Q}_{\rm B}e^{t\hat{\Phi}}),(e^{-t\hat{\Phi}}\hat{\eta}_0 e^{t\hat{\Phi}})\right\}\Big\rangle\Big\rangle, \tag{1.1}$$

where $\hat{\Phi}$ denotes a string field of NS sector which corresponds to a vertex operator of ghost number 0 and picture number 0 in the conformal field theory (CFT). In order to incorporate GSO($-$) sector into the theory on a BPS D-brane, we have to introduce internal Chan-Paton factors:

$$\hat{\Phi} = \Phi_+ \otimes \mathbf{1} + \Phi_- \otimes \sigma_1, \tag{1.2}$$

where the subscript $+$ ($-$) implies that the corresponding vertex operator is in the GSO($+$) (GSO($-$)) sector. The operators $\hat{Q}_{\rm B}$ and $\hat{\eta}_0$ are defined as

$$\hat{Q}_{\rm B} = Q_{\rm B}\otimes\sigma_3, \quad \hat{\eta}_0 = \eta_0\otimes\sigma_3, \tag{1.3}$$

where $Q_{\rm B}$ and η_0 are the ordinary operators without cocycle factors. The bracket $\langle\langle\cdots\rangle\rangle$ is defined by a CFT correlator in the large Hilbert space and a trace over internal Chan-Paton matrices. The action is invariant under the infinitesimal gauge transformation, $\delta e^{\hat{\Phi}} = (\hat{Q}_{\rm B}\delta\hat{\Lambda})*e^{\hat{\Phi}} + e^{\hat{\Phi}}*(\hat{\eta}_0\delta\hat{\Lambda}')$, where $\delta\hat{\Lambda}$ and $\delta\hat{\Lambda}'$ are infinitesimal parameters. Variating the action (1.1), we can derive the equation of motion to be

$$\hat{\eta}_0(e^{-\hat{\Phi}} * \hat{Q}_{\rm B}e^{\hat{\Phi}}) = 0. \tag{1.4}$$

This is the equation to be solved in this paper. For details of the definition, see for instance ref. [14].

This paper is organized as follows. In section 2, we will construct a tachyonic lump solution. In the bosonic case at the critical radius, there is an $su(2)$ current algebra which is useful to discuss descent relations of bosonic D-branes and to construct analytical lump solutions. At first, we find that a similar $su(2)$ supercurrent algebra exists even in the theory on a non-BPS D-brane. Using this supercurrent, we can solve the equation of motion and find an analytic lump solution. The vacuum energy of the resulting solution vanishes exactly as well as the BPS case. In section 3, we will discuss the theory expanded around the tachyonic lump solution. To interpret physical meanings of the expanded theory, fermionization of the compactified direction plays a key role, that is used to discuss a tachyonic lump in the context of boundary conformal field theory [19, 18]. Finally, we find that at the critical value of the solution the expanded theory is equivalent to the theory on a D-brane-anti-D-brane pair. Although this result is expected from boundary conformal field theory, we will provide a complete proof including interaction terms based on the analytic classical solution to eq. (1.4) in open superstring field theory. In section 4 we conclude with a brief summary and open problems.

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2. Tachyonic lump solutions

2.1 $su(2)$ supercurrent algebra

We compactify one of tangential directions to the brane on a circle of radius $R = \sqrt{2\alpha'}$. We take it as the 9-th direction and write the string coordinate as $X^9(z,\bar{z}) = (X^9(z)+X^9(\bar{z}))/2$ and its supersymmetric partner as $\psi^9(z)$. The operator product expansions (OPEs) of these fields are given by $X^9(y)X^9(z) \sim -2\alpha' \log(y-z)$ and $\psi^9(y)\psi^9(z) \sim 1/(y-z)$.

Similarly to the bosonic case, we can find a level one $su(2)$ supercurrent algebra at the critical radius, although the critical radius in the superstring case is inequivalent to that of bosonic case. The three supercurrents are given by

$$\mathbf{J}^1(z,\theta) = \sqrt{2}\sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)c_1 + \theta\sqrt{2}\psi^9\cos\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)c_2\,, \tag{2.1}$$

$$\mathbf{J}^2(z,\theta) = \sqrt{2}\cos\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)c_1 + \theta(-\sqrt{2})\psi^9\sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)c_2\,, \tag{2.2}$$

$$\mathbf{J}^3(z,\theta) = \psi^9(z)c_3 + \theta\frac{i}{\sqrt{2\alpha'}}\partial X^9(z)\,. \tag{2.3}$$

Here, we have introduced the cocycle factors c_i defined as

$$c_3^2 = 1, \quad c_i c_j = \delta_{ij} + i\epsilon_{ijk}c_k \quad i,j,k = 1,2,3, \quad \epsilon_{123} = +1\,, \tag{2.4}$$

where ϵ_{ijk} is the totally antisymmetric tensor. Writing $\mathbf{J}^a(z,\theta) \equiv \psi^a(z) + \theta J^a(z)$ $(a = 1,2,3)$, we obtain the following current algebra,

$$\psi^a(y)\psi^b(z) \sim (y-z)^{-1}\delta_{ab}\,, \tag{2.5}$$

$$J^a(y)\psi^b(z) \sim (y-z)^{-1}(-i\epsilon_{abc}\psi^c(z))\,, \tag{2.6}$$

$$J^a(y)J^b(z) \sim (y-z)^{-2}\delta_{ab} + (y-z)^{-1}(-i\epsilon_{abc}J^c(z))\,. \tag{2.7}$$

This is the same as $su(2)$ supercurrent algebra obtained by substituting $\Omega^{ab} = 2\delta_{ab}$ and $f^{ab}{}_c = -i\epsilon_{abc}$ in eqs. (3.1)–(3.3) in ref. [1].

From these supercurrents, we can construct the energy-momentum tensor by using the Sugawara method. First, we can find the following equations,

$$-:\psi^1\partial\psi^1:(z) = -\frac{1}{4\alpha'}(\partial X^9)^2(z) - \cos\left(\frac{2X^9}{\sqrt{2\alpha'}}\right)(z)\,, \tag{2.8}$$

$$-:\psi^2\partial\psi^2:(z) = -\frac{1}{4\alpha'}(\partial X^9)^2(z) + \cos\left(\frac{2X^9}{\sqrt{2\alpha'}}\right)(z)\,, \tag{2.9}$$

$$:J^1J^1:(z) = -\psi^9\partial\psi^9(z) + \cos\left(\frac{2X^9}{\sqrt{2\alpha'}}\right)(z) - \frac{1}{4\alpha'}(\partial X^9)^2(z)\,, \tag{2.10}$$

$$:J^2J^2:(z) = -\psi^9\partial\psi^9(z) - \cos\left(\frac{2X^9}{\sqrt{2\alpha'}}\right)(z) - \frac{1}{4\alpha'}(\partial X^9)^2(z)\,, \tag{2.11}$$

$$:\psi^a\psi^b:(z) = -i\epsilon_{abc}J^c(z) \qquad (a \neq b)\,, \tag{2.12}$$

$$:\psi^aJ^b:(z) - :J^a\psi^b:(z) = i\epsilon_{abc}\partial\psi^c(z)\,, \qquad (a \neq b)\,, \tag{2.13}$$

$$:J^1\psi^1:(z) = :J^2\psi^2:(z) = :J^3\psi^3:(z) = \frac{i}{\sqrt{2\alpha'}}\psi^9\partial X^9(z)c_3\,. \tag{2.14}$$

Then, we obtain the energy-momentum tensor and the world-sheet supercurrent as[2]

$$T^9(z) = \frac{1}{2} : (J^a J^a + \partial\psi^a\psi^a) : (z) - \frac{i}{6}\epsilon_{abc} : (J^a : \psi^b\psi^c : + \psi^a : (\psi^b J^c - J^b\psi^c) :) : (z)$$
$$= -\frac{1}{4\alpha'}(\partial X^9)^2(z) - \frac{1}{2}\psi^9\partial\psi^9(z)\,, \tag{2.15}$$
$$G^9(z) = : J^a\psi^a : (z) - \frac{i}{3}\epsilon_{abc} : \psi^a : \psi^b\psi^c :: (z) = \frac{i}{\sqrt{2\alpha'}}\psi^9\partial X^9(z)c_3\,. \tag{2.16}$$

Here, we should note that the world-sheet supercurrent $G^9(z)$ contains the cocycle factor c_3. Due to the presence of cocycle factors, the operators, $T^9(z)$, $G^9(z)$, $\psi^a(z)$ and $J^a(z)$, satisfy a superconformal current algebra with $c = 3/2$.

To incorporate the GSO(−) states into a string field, we have to introduce internal Chan-Paton indices as in ref. [5, 14]. In the theory on a non-BPS D-brane, fermionic operators like the BRST charge are tensored with a Pauli matrix as seen in eq. (1.3). Since the world-sheet supercurrent $G^9(z)$ is a fermionic operator and it is tensored with c_3, we identify the cocycle factors c_i with Pauli matrices representing the internal Chan-Paton factors:

$$c_3 = \sigma_3, \quad c_1 = \sigma_2, \quad c_2 = -\sigma_1. \tag{2.17}$$

We note that the $SU(2)$ symmetry is not realized on a non-BPS D-brane in spite of the fact that the $su(2)$ supercurrent algebra exists in the theory. In fact, $J_0^a = \oint \frac{dz}{2\pi i} J^a(z)$ is not to be a derivation with respect to the star product although $[\hat{Q}_{\rm B},\, J_0^a] = [\hat{\eta}_0,\, J_0^a] = 0$. Because we find that $J_0^a(\hat{\Psi}_1 * \hat{\Psi}_2) = (J_0^a\hat{\Psi}_1) * \hat{\Psi}_2 + ((-1)^{\hat{F}+\hat{n}}\hat{\Psi}_1) * (J_0^a\hat{\Psi}_2)$ for $a = 1, 2$ due to cocycle factors and quantized momentum along the 9-th direction, where $\hat{F}$ and $\hat{n}$ are operators counting fermion number and the 9-th momentum as defined later by (3.10) and (3.13). But, at the same time, $J_0^3(\hat{\Psi}_1 * \hat{\Psi}_2) = (J_0^3\hat{\Psi}_1) * \hat{\Psi}_2 + \hat{\Psi}_1 * (J_0^3\hat{\Psi}_2)$. Thus, the $SU(2)$ symmetry is broken to $U(1)$ by the interaction terms in the action (1.1).

2.2 Classical solutions via supercurrents

In ref. [1], we have constructed analytic classical solutions in the theory on BPS D-branes by means of supercurrent algebra. Now that we possess the supercurrent algebra including GSO(−) sector, we can apply the same method to the theory on the non-BPS D-branes. Taken $\mathbf{J}^1(z,\theta)$ as the supercurrent, the classical solution is given by

$$\hat{\Phi}_0 = -\tilde{V}_L(F)I\,, \tag{2.18}$$
$$\tilde{V}_L(F) = \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z)\tilde{v}(z), \tag{2.19}$$
$$\tilde{v}(z) = \frac{1}{\sqrt{2}}\, c\xi e^{-\phi}(z) \otimes \sigma_3 \times \psi^1(z), \tag{2.20}$$

where I is the identity string field and $C_{\rm left}$ denotes a counter-clockwise path along a half of the unit circle, i.e., $-\pi/2 < \sigma < \pi/2$ for $z = e^{i\sigma}$. $F(z)$ is a function on the unit circle

[2]We have used the definition in ref. [1].

$|z| = 1$ satisfying $F(-1/z) = z^2 F(z)$ [1].[3] We must impose an additional constraint on $F(z)$ due to the reality condition of the string field as in ref. [1]. The cocycle factor σ_3 should be attached in $\tilde{v}(z)$ since the ghost factor $c\xi e^{-\phi}(z)$ is Grassmann odd.[4] Substituting $\psi^1(z)$ of eq. (2.1) into eq. (2.20), the operator $\tilde{v}(z)$ is rewritten as[5]

$$\tilde{v}(z) = -i\, c\xi e^{-\phi} \sin\left(\frac{X^9(z)}{\sqrt{2\alpha'}}\right) \otimes \sigma_1. \tag{2.21}$$

It turns out that this classical solution represents a non-trivial configuration of the GSO(−) string field. Since the GSO(−) states include a tachyonic mode, this solution can be regarded as a kind of tachyonic lump solutions.

Now, we can easily find that the equation of motion actually holds. First, we define the operator $V_L(g)$ as

$$V_L(g) = \int_{C_{\rm left}} \frac{dz}{2\pi i} g(z) v(z), \tag{2.22}$$

$$v(z) = [\hat{Q}_{\rm B},\, \tilde{v}(z)] = \frac{1}{\sqrt{2}}(c(z) \otimes \sigma_3) J^1(z) + \frac{1}{\sqrt{2}} \eta e^{\phi} \psi^1(z). \tag{2.23}$$

For the operators V_L and $\tilde{V}_L$, we find the commutation relations

$$[\hat{Q}_{\rm B},\, \tilde{V}_L(g)] = V_L(g), \tag{2.24}$$

$$[\tilde{V}_L(g_1),\, V_L(g_2)] = -\frac{1}{2} C_L(g_1 g_2) \otimes \sigma_3, \tag{2.25}$$

where $C_L(g) \equiv \int_{\rm C_{left}} \frac{dz}{2\pi i} g(z) c(z)$. Then, taking into account of the properties of these operators associated with the star product [1], we can obtain

$$e^{-\hat{\Phi}_0} * \hat{Q}_{\rm B} e^{\hat{\Phi}_0} = (e^{\tilde{V}_L(F)} \hat{Q}_{\rm B} e^{-\tilde{V}_L(F)}) I = -V_L(F) I + \frac{1}{4} C_L(F^2) I \otimes \sigma_3. \tag{2.26}$$

The ξ zero mode is not contained in both operators $V_L(F)$ and $C_L(F^2)$ and the identity string field satisfies $\eta_0 I = 0$. As a result, we find that $\hat{\eta}_0(e^{-\hat{\Phi}_0} * \hat{Q}_{\rm B} e^{\hat{\Phi}_0}) = 0$ and the equation of motion holds.

Concerning the vacuum energy, we can obtain it by calculating the correlation function $\left\langle\!\!\left\langle (\hat{\eta}_0 \hat{\Phi}_0)(e^{-t\hat{\Phi}_0} \hat{Q}_{\rm B} e^{t\hat{\Phi}_0}) \right\rangle\!\!\right\rangle$. Similarly, it can be seen that there is no ξ zero mode in $e^{-t\hat{\Phi}_0} \hat{Q}_{\rm B} e^{t\hat{\Phi}_0}$ for arbitrary t. This fact is sufficient to show that the vacuum energy of the classical solution vanishes exactly in the same way as that of the GSO(+) solution in ref. [1].

[3] Under this condition, $F(z)$ cannot be a non-zero constant.

[4] We note that $e^{q\phi}$ (q : odd) is a fermionic operator. More precisely, we need a cocycle factor to represent statistical property of the operator.

[5] We have adjusted $c_1 = \sigma_2,\ c_2 = -\sigma_1$ in eq. (2.17) so that $\tilde{v}(z)$ has the cocycle factor σ_1. If we choose ψ^2 instead of ψ^1 in eq. (2.20), we get cosine-type solution. These sine and cosine type solutions are related by $U(1)$-symmetry, which is generated by J_0^3.

3. Superstring field theory around the solution

We consider the action expanded around the classical solution in order to provide physical interpretation of the solution. If we expand the string field as $e^{\hat{\Phi}} = e^{\hat{\Phi}_0} e^{\hat{\Phi}'}$, the action (1.1) becomes

$$S[\hat{\Phi}; \hat{Q}_{\rm B}] = S[\hat{\Phi}_0; \hat{Q}_{\rm B}] + S[\hat{\Phi}'; \hat{Q}'_{\rm B}]. \tag{3.1}$$

The first term of the right-hand side corresponds to the vacuum energy of the solution, which is seen to be zero as discussed above. Then, the expanded action takes the same form as the original action except that the BRST charge is changed depending on the classical solution. Accordingly, we will investigate the new BRST charge $\hat{Q}'_{\rm B}$ to determine the spectrum around the solution.

3.1 Fermionization and rebosonization

To find the spectrum around the classical solution, it is convenient to fermionize the scalar field $X^9(z)$ as in refs. [19, 18]:

$$e^{\pm\frac{i}{\sqrt{2\alpha'}}X^9(z)} = \frac{1}{\sqrt{2}}(\xi^9(z) \pm i\eta^9(z)) \otimes \tau_1, \tag{3.2}$$

where $\xi^9(z)$ and $\eta^9(z)$ are fermionic fields and the Pauli matrices τ_i denote cocycle factors. To ensure correct (anti-)commutation relations between various fields, we also attach a cocycle factor τ_3 to all other fermionic fields. For example, $\psi^9(z)$ is replaced with $\psi^9(z)\otimes\tau_3$, and the derivation operator $\hat{\eta}_0$ is written as $\hat{\eta}_0 = \eta_0 \otimes \sigma_3 \otimes \tau_3$. Using the fermionization rule (3.2), the supercurrents (2.1), (2.2) and (2.3) can be expressed as

$$\mathbf{J}^1(z,\theta) = \eta^9(z) \otimes \sigma_2 \otimes \tau_1 + \theta\,(-i\psi^9\xi^9(z)) \otimes \sigma_1 \otimes \tau_2\,, \tag{3.3}$$

$$\mathbf{J}^2(z,\theta) = \xi^9(z) \otimes \sigma_2 \otimes \tau_1 + \theta\,(i\psi^9\eta^9(z)) \otimes \sigma_1 \otimes \tau_2\,, \tag{3.4}$$

$$\mathbf{J}^3(z,\theta) = \psi^9(z) \otimes \sigma_3 \otimes \tau_3 + \theta\,(-i\xi^9\eta^9(z)) \otimes \mathbf{1} \otimes \mathbf{1}\,. \tag{3.5}$$

Similarly, the energy-momentum tensor (2.15) and the world-sheet supercurrent (2.16) are rewritten as

$$T^9(z) = \left(-\frac{1}{2}\xi^9\partial\xi^9(z) - \frac{1}{2}\eta^9\partial\eta^9(z) - \frac{1}{2}\psi^9\partial\psi^9(z)\right) \otimes \mathbf{1} \otimes \mathbf{1}\,, \tag{3.6}$$

$$G^9(z) = -i\xi^9\eta^9\psi^9(z) \otimes \sigma_3 \otimes \tau_3\,. \tag{3.7}$$

Then, the BRST charge is expressed as $\hat{Q}_{\rm B} = Q_{\rm B} \otimes \sigma_3 \otimes \tau_3$.

Since we compactify the 9-th direction to the circle, the momentum along this direction is quantized and it is labeled by even and odd integers. Applying the fermionization rule to the string field, the GSO(+) states with the odd momentum carry the cocycle factor τ_1. Since the GSO(−) states correspond to fermionic vertex operators, the cocycle factor τ_3 (τ_2) is attached to the GSO(−) states with even (odd) momentum. Then, we can express the string field as

$$\hat{\Phi} = \Phi^{\rm e}_+ \otimes \mathbf{1} \otimes \mathbf{1} + \Phi^{\rm o}_+ \otimes \mathbf{1} \otimes \tau_1 + \Phi^{\rm e}_- \otimes \sigma_1 \otimes \tau_3 + \Phi^{\rm o}_- \otimes \sigma_1 \otimes \tau_2, \tag{3.8}$$

where the subscript $\pm$ denotes GSO parity and the superscript e (o) implies the state with even (odd) momentum. For details, the world-sheet fermion number is defined as

$$(-1)^{\hat{F}}|\Phi_\pm\rangle = \pm|\Phi_\pm\rangle\,, \tag{3.9}$$

where the operator $\hat{F}$ in our convention is[6]

$$\hat{F} = \oint \frac{dz}{2\pi i}\left(\sum_{k=1}^{5} :\psi_+^k\psi_-^k: (z) - \partial\phi(z)\right), \tag{3.10}$$

$$\psi_\pm^1 \equiv \frac{i}{\sqrt{2}}(\psi^0 \pm \psi^1), \quad \psi_\pm^k \equiv \frac{1}{\sqrt{2}}(\psi^{2k-2} \pm i\psi^{2k-1}), \quad k = 2,3,4,5. \tag{3.11}$$

The momentum parity is defined as

$$(-1)^{\hat{n}}|\Phi^{\rm e}\rangle = +|\Phi^{\rm e}\rangle\,, \qquad (-1)^{\hat{n}}|\Phi^{\rm o}\rangle = -|\Phi^{\rm o}\rangle\,, \tag{3.12}$$

where the operator $\hat{n}$ counting the 9-th momentum is given by

$$\hat{n} = \oint \frac{dz}{2\pi i}\frac{i}{\sqrt{2\alpha'}}\partial X^9(z) = \oint \frac{dz}{2\pi i} i\eta^9\xi^9(z)\,. \tag{3.13}$$

In general, the string field of an even ghost number is expanded by the same cocycle factors. The string field of an odd ghost number, like gauge transformation parameters, can be written as

$$\hat{\Lambda} = \Phi_+^{\rm e}\otimes\sigma_3\otimes\tau_3 + \Phi_+^{\rm o}\otimes\sigma_3\otimes\tau_2 + \Phi_-^{\rm e}\otimes\sigma_2\otimes\mathbf{1} + \Phi_-^{\rm o}\otimes\sigma_2\otimes\tau_1. \tag{3.14}$$

We can find another representation of the conformal field theory for (ψ^9, ξ^9, η^9) by the rebosonization [19, 18]

$$(\xi^9(z) \pm i\psi^9(z)) = \sqrt{2}e^{\pm\frac{i}{\sqrt{2\alpha'}}\phi^9(z)}\otimes\tilde{\tau}_1, \tag{3.15}$$

where the Pauli matrices $\tilde{\tau}_i$ are cocycle factors and we assign the cocycle $\tilde{\tau}_3$ to fermionic fields except $\psi^9(z)$ and $\xi^9(z)$. We can easily rewrite all operators and the string field using the bosonization rule (3.15). In particular, the supercurrents (3.3), (3.4) and (3.5) are expressed as

$$\mathbf{J}^1(z,\theta) = \eta^9\otimes\sigma_2\otimes\tau_1\otimes\tilde{\tau}_3 - \theta\frac{i}{\sqrt{2\alpha'}}\partial\phi^9(z)\otimes\sigma_1\otimes\tau_2\otimes\mathbf{1}, \tag{3.16}$$

$$\mathbf{J}^2(z,\theta) = \sqrt{2}\cos\left(\frac{\phi^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_2\otimes\tau_1\otimes\tilde{\tau}_1 + \theta\sqrt{2}\eta^9\sin\left(\frac{\phi^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_1\otimes\tau_2\otimes\tilde{\tau}_2, \tag{3.17}$$

$$\mathbf{J}^3(z,\theta) = \sqrt{2}\sin\left(\frac{\phi^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_3\otimes\tau_3\otimes\tilde{\tau}_1 - \theta\sqrt{2}\eta^9\cos\left(\frac{\phi^9}{\sqrt{2\alpha'}}\right)(z)\otimes\mathbf{1}\otimes\mathbf{1}\otimes\tilde{\tau}_2. \tag{3.18}$$

We note that we have to change the normalization of the action if we apply the fermionization or the rebosonization to the string field. In the action (1.1), we take the trace of

[6]Here ϕ is a bosonized ghost coming from $\gamma = \eta e^{\phi}$, $\beta = e^{-\phi}\partial\xi$, and ψ^μ ($\mu = 0, 1, \cdots, 9$) are matter fermions. The reader should not confuse them with ϕ^9 in eq. (3.15) and the lowest components ψ^a of the $su(2)$ supercurrent $\mathbf{J}^a(z,\theta)$ ($a = 1, 2, 3$).

all Chan-Paton indices. If we fermionize X^9, the Chan-Paton factors τ_i with their trace produce an extra factor of two for the action. Consequently, we must divide the action by two in order to provide the same action for the component fields. Furthermore, if we rebosonize and introduce the additional Chan-Paton factors $\tilde{\tau}_i$, we need to divide the action by four.

3.2 The theory expanded around the solution

The new BRST operator in the expanded action $S[\hat{\Phi}'; \hat{Q}'_{\rm B}]$ in (3.1) around a solution $\hat{\Phi}_0$ to the equation of motion (1.4) is generically expressed as

$$\hat{Q}'_{\rm B}\hat{\Psi} = \hat{Q}_{\rm B}\hat{\Psi} + \hat{A}_0 * \hat{\Psi} - (-)^{{\rm gh}(\hat{\Psi})}\hat{\Psi} * \hat{A}_0, \quad \hat{A}_0 = e^{-\hat{\Phi}_0} * \hat{Q}_{\rm B} e^{\hat{\Phi}_0} \quad \text{for } {}^\forall\hat{\Psi}. \tag{3.19}$$

This formula can be derived as appendix B in ref. [1], for example, because algebraic relations are almost the same as the original GSO projected theory [14]. Here ${\rm gh}(\hat{\Psi})$ denotes ghost number of $\hat{\Psi}$ and it is counted by $n_{\rm gh} = -\oint \frac{dz}{2\pi i}(:bc: + :\xi\eta:)$. A string field $\hat{\Psi}$ takes the form of (3.8) for even ghost number and (3.14) for odd ghost number. The sign factor $(-)^{{\rm gh}(\hat{\Psi})}$ instead of "Grassmannality" appears because $\hat{Q}'_{\rm B}$ should be an anti-derivation as original $\hat{Q}_{\rm B}$ in the sense that $\hat{Q}'_{\rm B}(\hat{\Psi}_1 * \hat{\Psi}_2) = (\hat{Q}'_{\rm B}\hat{\Psi}_1) * \hat{\Psi}_2 + (-)^{{\rm gh}(\hat{\Psi}_1)}\hat{\Psi}_1 * (\hat{Q}'_{\rm B}\hat{\Psi}_2)$ [20].

We rewrite the operator $\tilde{v}(z)$ in the solution (2.18) by using the fermionic fields (ψ^9, ξ^9, η^9) through the fermionization rule (3.2):

$$\tilde{v}(z) = \frac{1}{\sqrt{2}} c\xi e^{-\phi}\, \eta^9(z) \otimes \sigma_1 \otimes \tau_2. \tag{3.20}$$

The operator (2.23) can be written as

$$v(z) = \left(\frac{-i}{\sqrt{2}} c\psi^9\xi^9(z) + \frac{1}{\sqrt{2}}\eta e^{\phi}\eta^9(z)\right) \otimes \sigma_2 \otimes \tau_1. \tag{3.21}$$

The operator $\hat{Q}'_{\rm B}$ (3.19) for the solution (3.20) can be found as

$$\hat{Q}'_{\rm B} = (Q_{\rm B} + \frac{1}{4}C(F^2)) \otimes \sigma_3 \otimes \tau_3 - V_L(F) - (-1)^{\hat{F}+\hat{n}} V_R(F), \tag{3.22}$$

where $C(F^2) = C_L(F^2) + C_R(F^2)$, $C_R(g) \equiv \int_{C_{\rm right}} \frac{dz}{2\pi i} g(z)c(z)$, $V_R(g) \equiv \int_{C_{\rm right}} \frac{dz}{2\pi i} g(z)v(z)$, $C_{\rm right}$ is a counter-clockwise path along a half of the unit circle:($|z| = 1$, ${\rm Re}\, z < 0$) as in ref. [1], and the operators $\hat{F}$ and $\hat{n}$ are given by eqs. (3.10) and (3.13). The extra sign factor $(-1)^{\hat{F}+\hat{n}}$ in front of $V_R(F)$ comes from exchange of order of $\hat{\Psi}$ and $v(z)$ in eq. (3.19). This new BRST operator can be rewritten in terms of a similarity transformation from the original operator,

$$\hat{Q}'_{\rm B} = e^{\tilde{V}_L(F) + (-1)^{\hat{F}+\hat{n}}\tilde{V}_R(F)}\, \hat{Q}_{\rm B}\, e^{-\tilde{V}_L(F) - (-1)^{\hat{F}+\hat{n}}\tilde{V}_R(F)}. \tag{3.23}$$

We notice that this relation cannot be used for a field redefinition in the expanded action because of $[\hat{\eta}_0, \tilde{V}_L(F) + (-1)^{\hat{F}+\hat{n}}\tilde{V}_R(F)] \neq 0$.

Furthermore, we can find another expression of the new BRST charge in terms of (ϕ^9, η^9). If the new BRST charge acts on the state of $(-1)^{\hat{F}+\hat{n}} = +1$, it becomes

$$\hat{Q}'_{\rm B} = e^{-\frac{i}{2\sqrt{\alpha'}}(\phi^9_L(F) + \phi^9_R(F)) \otimes \sigma_1 \otimes \tau_2}\, \hat{Q}_{\rm B}\, e^{\frac{i}{2\sqrt{\alpha'}}(\phi^9_L(F) + \phi^9_R(F)) \otimes \sigma_1 \otimes \tau_2}, \tag{3.24}$$

and for the case of $(-1)^{\hat{F}+\hat{n}} = -1$,

$$\hat{Q}'_{\rm B} = e^{-\frac{i}{2\sqrt{\alpha'}}(\phi^9_L(F)-\phi^9_R(F))\otimes\sigma_1\otimes\tau_2}\,\hat{Q}_{\rm B}\,e^{\frac{i}{2\sqrt{\alpha'}}(\phi^9_L(F)-\phi^9_R(F))\otimes\sigma_1\otimes\tau_2}, \tag{3.25}$$

where $\phi^9_{L/R}(F) \equiv \int_{C_{\rm left/right}} \frac{dz}{2\pi i} F(z)\phi^9(z)$. They are derived from the direct calculation or from the expression (3.23) and the following anti-commutation relation,

$$\{\hat{Q}_{\rm B},\, \Omega_{L/R}(F)\} = 2\sqrt{\alpha'}\tilde{V}^1_{L/R}(F) + i\phi^9_{L/R}(F)\otimes\sigma_1\otimes\tau_2\,, \tag{3.26}$$

$$\Omega_{L/R}(F) \equiv -\int_{C_{\rm left/right}} \frac{dz}{2\pi i} F(z)\, i\, c\xi\partial\xi e^{-2\phi}\,\phi^9(z)\otimes\sigma_2\otimes\tau_1. \tag{3.27}$$

Noting $[\hat{\eta}_0, \phi^9_{L/R}(F)\otimes\sigma_1\otimes\tau_2] = 0$, these expressions given in eqs. (3.24) and (3.25) for the new BRST operator imply that the expanded action around the solution can be transformed back to the original action by the string field redefinition,

$$\hat{\Phi}'' = e^{\frac{i}{2\sqrt{\alpha'}}\phi^9_L(F)I\otimes\sigma_1\otimes\tau_2} * \hat{\Phi}' * e^{-\frac{i}{2\sqrt{\alpha'}}\phi^9_L(F)I\otimes\sigma_1\otimes\tau_2}. \tag{3.28}$$

Actually, this string field redefinition does not change the interaction terms in the action and, depending on the $(-1)^{\hat{F}+\hat{n}}$ parity of the string field, the redefinition can be rewritten as

$$\hat{\Phi}'' = \begin{cases} e^{\frac{i}{2\sqrt{\alpha'}}(\phi^9_L(F)+\phi^9_R(F))\otimes\sigma_1\otimes\tau_2}\hat{\Phi}' & \text{for } \ (-1)^{\hat{F}+\hat{n}} = +1 \\ e^{\frac{i}{2\sqrt{\alpha'}}(\phi^9_L(F)-\phi^9_R(F))\otimes\sigma_1\otimes\tau_2}\hat{\Phi}' & \text{for } \ (-1)^{\hat{F}+\hat{n}} = -1. \end{cases} \tag{3.29}$$

The difference in sign in the right-hand side arises from (anti-)commutation relations of Chan-Paton factors. For the string field (3.8), the $(-1)^{\hat{F}+\hat{n}} = +1$ sector involves cocycle factors $\mathbf{1}\otimes\mathbf{1}$ and $\sigma_1\otimes\tau_2$, which commute with the generator $\sigma_1\otimes\tau_2$ of the string field redefinition. However, the generator anti-commutes with the cocycle factors $\mathbf{1}\otimes\tau_1$ and $\sigma_1\otimes\tau_3$ and then the minus sign appears for the $(-1)^{\hat{F}+\hat{n}} = -1$ sector, $\Phi^{\rm o}_+$ and $\Phi^{\rm e}_-$ in eq. (3.8).

Though the expanded action is transformed to the original one, the string field redefinition has a physical effect. As discussed for the case of the Wilson line solution in ref. [1], the spectrum is changed from that of the original theory due to the zero-mode of the operator $\phi^9(z)$. We have no zero-mode in the operator $\phi^9(F) \equiv \phi^9_L(F) + \phi^9_R(F)$ because we impose the condition $F(-1/z) = z^2F(z)$ in the solution (2.18) and then the coefficients of the zero-mode cancel as

$$\int_{C_{\rm left}} \frac{dz}{2\pi i} F(z) + \int_{C_{\rm right}} \frac{dz}{2\pi i} F(z) = 0\,, \tag{3.30}$$

whereas $\phi^9_\Delta(F) \equiv \phi^9_L(F) - \phi^9_R(F)$ includes the zero-mode. As a result, the $(-1)^{\hat{F}+\hat{n}} = -1$ sector is multiplied by the extra factor,

$$\exp\left(i\frac{f}{\sqrt{\alpha'}}\,\hat{\phi}^9_0\otimes\sigma_1\otimes\tau_2\right), \quad f \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z), \tag{3.31}$$

where $\hat{\phi}_0^9$ denotes the zero-mode operator of $\phi^9(z)$. This zero-mode factor changes the momentum of the string field along the ϕ^9 direction as $p_{\phi^9} \to p_{\phi^9} \pm f/\sqrt{\alpha'}$. The momentum shift only for the $(-1)^{\hat{F}+\hat{n}} = -1$ sector is exactly the same effect as that of a tachyonic lump solution as discussed in the context of boundary conformal field theory [18]. Hence, our analytic solution (2.18) represents the same tachyonic lump solution in open superstring field theory, and the half integration mode, f, corresponds to the Wilson line along the ϕ^9 direction.

3.3 The expanded theory at the critical value of f

We discuss a tachyonic lump solution corresponding to the critical value of f in (3.31), namely

$$f = \frac{2m+1}{\sqrt{2}}, \quad m \in \mathbf{Z}. \tag{3.32}$$

At the critical value, the redefined field $\hat{\Phi}''$ (3.29) can be rewritten again by the fermionic fields (ψ^9, ξ^9, η^9) instead of (ϕ^9, η^9). Moreover, we can write its string field by the original string coordinates (X^9, ψ^9) through the rebosonization of (ξ^9, η^9) to X^9.

Using the fermionic fields (ψ^9, ξ^9, η^9), we can write the redefined field as

$$\begin{aligned}
\hat{\Phi}'' &= e^{\frac{i}{2\sqrt{\alpha'}}\phi^9(F)\otimes\sigma_1\otimes\tau_2}(\Phi_+^{\mathrm{e}} \otimes \mathbf{1} \otimes \mathbf{1} + \Phi_-^{\mathrm{o}} \otimes \sigma_1 \otimes \tau_2) \\
&\quad + e^{\frac{i}{2\sqrt{\alpha'}}\phi_\Delta^9(F)\otimes\sigma_1\otimes\tau_2}(\Phi_+^{\mathrm{o}} \otimes \mathbf{1} \otimes \tau_1 + \Phi_-^{\mathrm{e}} \otimes \sigma_1 \otimes \tau_3) \\
&= \left(\cos\left(\frac{\phi^9(F)}{2\sqrt{\alpha'}}\right)\Phi_+^{\mathrm{e}} + i\sin\left(\frac{\phi^9(F)}{2\sqrt{\alpha'}}\right)\Phi_-^{\mathrm{o}}\right) \otimes \mathbf{1} \otimes \mathbf{1} \\
&\quad + \left(i\sin\left(\frac{\phi^9(F)}{2\sqrt{\alpha'}}\right)\Phi_+^{\mathrm{e}} + \cos\left(\frac{\phi^9(F)}{2\sqrt{\alpha'}}\right)\Phi_-^{\mathrm{o}}\right) \otimes \sigma_1 \otimes \tau_2 \\
&\quad + \left(\cos\left(\frac{\phi_\Delta^9(F)}{2\sqrt{\alpha'}}\right)\Phi_+^{\mathrm{o}} - \sin\left(\frac{\phi_\Delta^9(F)}{2\sqrt{\alpha'}}\right)\Phi_-^{\mathrm{e}}\right) \otimes \mathbf{1} \otimes \tau_1 \\
&\quad + \left(\sin\left(\frac{\phi_\Delta^9(F)}{2\sqrt{\alpha'}}\right)\Phi_+^{\mathrm{o}} + \cos\left(\frac{\phi_\Delta^9(F)}{2\sqrt{\alpha'}}\right)\Phi_-^{\mathrm{e}}\right) \otimes \sigma_1 \otimes \tau_3.
\end{aligned} \tag{3.33}$$

Here, the operators, $\cos(\phi_{(\Delta)}^9(F)/2\sqrt{\alpha'})$ and $\sin(\phi_{(\Delta)}^9(F)/2\sqrt{\alpha'})$, are represented in terms of the fermionic fields ξ^9 and ψ^9 instead of ϕ^9 thanks to (3.15) and $\partial\phi^9 = -\sqrt{2\alpha'}\xi^9\psi^9$.

At first, we consider the operators $\cos(\phi_\Delta^9(F)/2\sqrt{\alpha'})$ and $\sin(\phi_\Delta^9(F)/2\sqrt{\alpha'})$. When we introduce the operator counting ϕ^9 momenta as $\hat{n}_{\phi^9} = \oint \frac{dz}{2\pi i} i\partial\phi^9/\sqrt{2\alpha'}$, the operators appeared in the redefinition have $(-1)^{\hat{n}_{\phi^9}} = -1$ because they carry the ϕ^9 momentum $p_{\phi^9} = f/\sqrt{\alpha'} = (2m+1)/\sqrt{2\alpha'}$. In addition, as in refs. [18, 19], we define the operator $\hat{F}_{\phi^9}$ counting the fermion number of η^9 and other spectator fermions, ψ^μ $(\mu = 0, 1, \cdots, 8)$ and $e^{q\phi}$ (q:odd). With this definition, the operators have $(-1)^{\hat{F}_{\phi^9}} = +1$. Then, we find that the operators have $(-1)^{\hat{F}_{\phi^9}+\hat{n}_{\phi^9}} = -1$.

Next, we consider the original fermion number and momenta of the operators. If we change the sign of ϕ^9, the fermions ξ^9, ψ^9 are transformed as

$$\xi^9 \to \xi^9, \quad \psi^9 \to -\psi^9, \tag{3.34}$$

because ϕ^9 is related to ξ^9 and ψ^9 through the rebosonization rule (3.15). Therefore we can determine the $(-1)^{\hat{F}}$ parity of some operators by means of the parity transformation of ϕ^9. We can find that $\cos(\phi^9_\Delta(F)/2\sqrt{\alpha'})$ has $(-1)^{\hat{F}} = +1$ and $\sin(\phi^9_\Delta(F)/2\sqrt{\alpha'})$ has $(-1)^{\hat{F}} = -1$. As discussed in ref. [18], we have the relation:

$$(-1)^{\hat{F}}(-1)^{\hat{n}} = (-1)^{\hat{F}_{\phi^9}}(-1)^{\hat{n}_{\phi^9}}. \tag{3.35}$$

Combining these results, we can determine the values of $(-1)^{\hat{F}}$ and $(-1)^{\hat{n}}$ individually for these operators. The resulting parities for operators are listed in the following table:

	$(-1)^{\hat{F}}$	$(-1)^{\hat{n}}$	$(-1)^{\hat{F}_{\phi^9}}$	$(-1)^{\hat{n}_{\phi^9}}$
$\cos\left(\frac{\phi^9_\Delta(F)}{2\sqrt{\alpha'}}\right)$	+	−	+	−
$\sin\left(\frac{\phi^9_\Delta(F)}{2\sqrt{\alpha'}}\right)$	−	+	+	−

(3.36)

Based on a similar consideration, we have the following results for other operators:

	$(-1)^{\hat{F}}$	$(-1)^{\hat{n}}$	$(-1)^{\hat{F}_{\phi^9}}$	$(-1)^{\hat{n}_{\phi^9}}$
$\cos\left(\frac{\phi^9(F)}{2\sqrt{\alpha'}}\right)$	+	+	+	+
$\sin\left(\frac{\phi^9(F)}{2\sqrt{\alpha'}}\right)$	−	−	+	+

(3.37)

From these results, we find that the first and second terms in eq. (3.33) have $(-1)^{\hat{F}} = +1$ and $(-1)^{\hat{n}} = +1$ and the third and fourth have $(-1)^{\hat{F}} = -1$ and $(-1)^{\hat{n}} = -1$, and then all components in the redefined string field (3.33) have $(-1)^{\hat{F}+\hat{n}} = +1$. Before the redefinition, the fields with $\mathbf{1} \otimes \tau_1$ and $\sigma_1 \otimes \tau_3$ have $(-1)^{\hat{F}+\hat{n}} = -1$. The parity of these states is changed after the redefinition. Alternatively, the quantum number of $\hat{F} + \hat{n}$ can be regarded as the fermion number assigning $+1$ to the fields ψ^9, ξ^9, η^9 in the fermionic representation. Consequently, with the fermionic representation, the statistical property of the fields with $\mathbf{1} \otimes \tau_1$ and $\sigma_1 \otimes \tau_3$ are changed under the string field redefinition. In order to ensure correct (anti-)commutation relations, we have to assign a cocycle factor of $\tilde{\tau}_1$ to these fields, and assign a cocycle factor $\tilde{\tau}_3$ to the derivations $\hat{Q}_{\rm B}$ and $\hat{\eta}_0$. After all, the redefined string field can be expressed using the fermionic representation as

$$\hat{\Phi}'' = \Psi^{\rm e}_+ \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} + \Psi'^{\rm e}_+ \otimes \mathbf{1} \otimes \tau_1 \otimes \tilde{\tau}_1 + \Psi^{\rm o}_- \otimes \sigma_1 \otimes \tau_3 \otimes \tilde{\tau}_1 + \Psi'^{\rm o}_- \otimes \sigma_1 \otimes \tau_2 \otimes \mathbf{1}. \tag{3.38}$$

Now, let us express the string field (3.38) in terms of the fields (X^9, ψ^9) through the rule (3.2). When we rebosonize (ξ^9, η^9) to X^9, we have to assign a cocycle factor τ_1 to states with an odd momentum and retain the cocycle factors τ_i under the earlier fermionization. According to this procedure, the string fields $\Psi^{\rm o}_-$ and $\Psi'^{\rm o}_-$ acquire an additional cocycle factor of τ_1 under the rebosonization. Hence, with the fields (X^9, ψ^9), the string field can be rewritten as

$$\hat{\Phi}'' = \Psi^{\rm e}_+ \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} + \Psi'^{\rm e}_+ \otimes \mathbf{1} \otimes \tau_1 \otimes \tilde{\tau}_1 + \Psi^{\rm o}_- \otimes \sigma_1 \otimes \tau_2 \otimes \tilde{\tau}_1 + \Psi'^{\rm o}_- \otimes \sigma_1 \otimes \tau_3 \otimes \mathbf{1}. \tag{3.39}$$

The derivations are expressed as same as before:

$$\hat{Q}_{\rm B} = Q_{\rm B} \otimes \sigma_3 \otimes \tau_3 \otimes \tilde{\tau}_3, \quad \hat{\eta}_0 = \eta_0 \otimes \sigma_3 \otimes \tau_3 \otimes \tilde{\tau}_3. \tag{3.40}$$

Here, we write the cocycle factors appeared in the string field (3.39) and the derivations (3.40) as

$$\Sigma_3 = \mathbf{1} \otimes \tau_1 \otimes \tilde{\tau}_1, \quad \Sigma_1 = \sigma_1 \otimes \tau_2 \otimes \tilde{\tau}_1, \quad \Sigma_2 = \sigma_1 \otimes \tau_3 \otimes \mathbf{1}, \quad \sigma = \sigma_3 \otimes \tau_3 \otimes \tilde{\tau}_3. \tag{3.41}$$

These matrices satisfy the following relations:

$$\Sigma_1^2 = \Sigma_2^2 = \Sigma_3^2 = \sigma^2 = 1\,, \tag{3.42}$$

$$[\sigma, \Sigma_3] = \{\sigma, \Sigma_1\} = \{\sigma, \Sigma_2\} = 0\,, \tag{3.43}$$

$$\Sigma_i \Sigma_j = i\epsilon_{ijk}\Sigma_k\,, \quad (i \neq j)\,. \tag{3.44}$$

We can represent the same algebra by the alternative Pauli matrices σ_i and τ_i:

$$\Sigma'_1 = \sigma_1 \otimes \tau_1, \quad \Sigma'_2 = \sigma_1 \otimes \tau_2, \quad \Sigma'_3 = \mathbf{1} \otimes \tau_3, \quad \sigma' = \sigma_3 \otimes \mathbf{1}. \tag{3.45}$$

Therefore, we can identify (Σ'_i, σ') with (Σ_i, σ) if we divide the action by two to compensate their different normalization.

Finally, under the above identification, we can represent the redefined string field in terms of (X^9, ψ^9) as

$$\hat{\Phi}'' = \Psi^{\rm e}_+ \otimes \mathbf{1} \otimes \mathbf{1} + \Psi'^{\rm e}_+ \otimes \mathbf{1} \otimes \tau_3 + \Psi^{\rm o}_- \otimes \sigma_1 \otimes \tau_1 + \Psi'^{\rm o}_- \otimes \sigma_1 \otimes \tau_2, \tag{3.46}$$

and the derivations as

$$\hat{Q}_{\rm B} = Q_{\rm B} \otimes \sigma_3 \otimes \mathbf{1}, \quad \hat{\eta}_0 = \eta_0 \otimes \sigma_3 \otimes \mathbf{1}. \tag{3.47}$$

The resulting string field theory including this string field and the derivations is exactly the same theory on a D-brane-anti-D-brane pair discussed in ref. [14], in which σ_i are the internal Chan-Paton indices to include the GSO$(-)$ sector and τ_i correspond to the conventional Chan-Paton indices introduced for a pair of branes. The string fields connecting a D-brane and an anti-D-brane have odd momenta along the X^9 direction, and the string fields attached both ends of the string to a single brane have even momenta. Consequently, the string field theory at the critical value of f describes the D-brane-anti-D-brane system in which a D-brane and an anti-D-brane are situated at antipodal points along the circle with the critical radius in the T-dual picture.

Thus, we find that the tachyonic lump solution corresponding to the critical value of f changes the theory on a single non-BPS D-brane to that of a D-brane-anti-D-brane pair. This is physically the same result obtained before in terms of boundary conformal field theory [18]. But, we should comment on a superficial difference between these results. In our case, the resulting branes are put on the X^9 direction, while in ref. [18] the branes are on the direction represented by $\phi'^9(z)$, which is another bosonic field given by a rebosonization of (η^9, ψ^9). As discussed in the previous section, the theory possesses the $su(2)$ supercurrent

algebra but the $SU(2)$ symmetry is broken on a non-BPS D-brane. However, it turns out that the $SU(2)$ symmetry is restored in the NS sector of the theory with the critical value of f and the bosonic coordinates (X^9, ϕ^9, ϕ'^9) can be rotated under this symmetry because all sectors have $(-1)^{\hat{F}+\hat{n}} = +1$ after the string field redefinition around the solution. Therefore, the difference is resolved by the $SU(2)$ rotation of X^9 to ϕ'^9.

4. Concluding remarks

We constructed the analytic classical solution in superstring field theory on the non-BPS D-brane in which the one direction X^9 is compactified to a circle with the critical radius. The solution corresponds to the tachyonic lump solution which corresponds to the Wilson line along the ϕ^9 direction. The vacuum energy of the solution vanishes exactly as that of the BPS case. At the critical value of f, the theory expanded around the solution is equivalent to the theory on a D-brane-anti-D-brane pair, including the interaction terms. These results agree with the facts expected from boundary conformal field theory. The $su(2)$ supercurrent algebra was useful for the analyses of the solution.

In ref. [1], we found some features of the solution on BPS D-branes. The solution has a well-defined Fock space expression and the half integration mode f is invariant under a class of gauge transformations in superstring field theory but other modes are not. Employing the same technique in ref. [1], we can easily find that the same is true in the case of non-BPS D-branes.

We should discuss the Ramond sector, which was out of the scope of this paper, to complete the correspondence of our solution to the tachyonic lump. The action on non-BPS D-branes including the Ramond sector is supposed to be constructed by extending the action on BPS D-branes given by ref. [21]. In the extended theory including the Ramond sector, our solution will satisfy the equation of motion. The problem is whether the string field redefinition, especially at the critical value of f, reproduces the expected result of the Wilson line along the ϕ^9 direction. It seems complicated to incorporate GSO$(-)$ states in the Ramond sector and assign appropriate cocycle factors consistently.

We can apply our method constructing the analytical solution to other cases of marginal deformations; a solution on non-BPS D-branes on an orbifold [18] and a vortex solution on a D-brane-anti-D-brane pair [19]. To realize marginal deformations, we have to take the critical radius of the compactified direction and the vacuum energy is always to be zero for these cases. If we deform the radius away from the critical value, we may be able to find more general solutions with a non-trivial vacuum energy. This problem is interesting because such a general solution may teach us how the closed string moduli changing the radius includes in open string field.

In this paper we show that there exists an analytic solution taking the value in the GSO$(-)$ sector. This fact indicates the possible existence of the analytic tachyon vacuum solution, at which non-BPS D-branes completely disappear, in open superstring field theory. If we find the analytic solution, we could prove the non-existence of open strings and the exact cancellation of the vacuum energy, as discussed in the bosonic case [22, 24, 23]. We

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expect that the evaluation of the vacuum energy in the supersymmetric case sheds lights on the problem what sort of regularization should be applied to the bosonic theory.

Acknowledgements

The authors would like to thank Yuji Igarashi and Katsumi Itoh for useful discussions and the Yukawa Institute for Theoretical Physics at Kyoto University for providing a stimulating atmosphere. Discussions during the YITP workshop YITP-W-05-08 on "String Theory and Quantum Field Theory" were useful to complete this work. They are also grateful to "Summer Institute String Theory 2005" at Sapporo during which part of this work was done. I.K. wishes to express his gratitude to Seiji Terashima for valuable comments.

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